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Survey Paper on Hilbert Transform With its Applications in Signal Processing

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Abstract- **In signal processing, Hilbert transform is used as an analytical tool. Hilbert Transform (HT) is useful in certain kinds of modulation schemes and also for bounded signals. The overall narrow band signals can be easily interpreted in terms of amplitude and frequency modulation. The various applications are present in magnetic recording, filter designing, edge detection and image processing.**

Keywords—Hilbert Transform; Bounded signals; magnetic recording; FourierTransform.

INTRODUCTION

Hilbert transform (HT) have played a very important role in signal and network theory and it also has very practical applications in various fields. The fields may also include communication systems, RADAR systems and medical imaging. HT provides a ±90° phase shift to the input signal, so if we choose the input signal to be a sine function then calculating its Hilbert transform will give cosine function. The HT is just like any of the special phase adjusted filters that are possible. In Fourier transform we change the time domain signal to the frequency domain signals, but in Hilbert transform domain of operation remains the same.

Hilbert transform can be used to design digital filters which can be infinite impulse response (IIR) and Finite impulse response (FIR) filters. In FIR filters there is no feedback while in IIR filters the feedback is present. HT acts as a causal sequence and relates the real part of Fourier transform to the imaginary part of Fourier transform. The Fourier Transforms require complete knowledge of both Real and Imaginary parts of the magnitude and phase for all frequencies in the range $-\pi < \omega < \pi$. Hilbert Transforms applied to causal signals takes advantage of the fact that Real sequences have Symmetric Fourier transforms.

The present paper is a survey of Hilbert transform with its generalized mathematical formulations that can be used further to perform the Hilbert transform of an input signal. This transform is quite different from rest of the transforms that are used in signal processing such as Fourier transform and Laplace transform, etc. In this paper we present the various applications in which HT can be used.

HILBERT TRANSFORM

Hilbert transform (HT) is an analytical tool that is useful for the representation of certain kinds of signals such as band pass signals. This transform is also used for various kinds of modulation schemes as series side band AM modulation.

HT is different from any other transform that is used in signal processing because in this no domain change is required. If we are taking the input signal to be in the time domain then by using the HT we get the output signal in time domain only. This special property of HT is true for frequency domain signal also, which further helps in the complexity reduction. Now we take the generalized form of the HT in equation (1).

$$x^\wedge(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}d\tau \qquad (1)$$

Where x^(t) is the Hilbert transform of x(t).

The HT involves the convolution of the input signal and the impulse response. Most importantly we take the transfer function of the Linear Filter as it satisfies superposition principle and it can be only represented in transfer function form. This linear filter will work to phase shift all frequency components by $-\pi/2$ radians. The magnitude characteristics of the filter are 1 for all frequencies whereas the real signals have positive as well as negative frequencies. As HT introduces a 90° phase shift twice causes a 180° phase shift, which can cause a phase reversal of the original signal.

The considered amplitudes of all frequency components in the signal, however are unaffected by transmission through the device. Such an ideal device is referred as Hilbert transformers. And for the inverse Hilbert transform (IHT) as shown in equation (2).

$$x(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x^\wedge(\tau)}{t-\tau}d\tau \qquad (2)$$

The equation (1) and (2) also shows that HT is a linear operator. The input and the output x(t) is termed as Hilbert transform pairs.

APPLICATIONS OF HILBERT TRANSFORM

Hilbert transform has got a wide range of applications in the analysis of system design. This transform is therefore useful for diverse purposes such as latency analysis in neuro-physiological signals, design of bizarre stimuli for psychoacoustic experiments, speech data compression for communication, regularization of convergence problems in multi-channel acoustic echo cancellation, and signal processing for auditory prostheses.

Here we are studying in detail about the three important areas in which HT can be used and implemented to get the desired results.

A. HT in Magnetic Recording

Hilbert transform in magnetic recording is conceptually based on the simultaneous use of magnetic charge and electric current. So we generalize the 2 dimensional (2D) HT to the 3D magnetic fields that come from the recording media. In the various areas of signal processing generalized HT is used for higher dimensions which is mathematical in nature and thus deals with scalar or analytic signals that are defined on multidimensional spaces.

Now this can be easily done by relating the longitudinal and transverse nature of the magnetic fields. And for this we use the 1D Hilbert transform. For both longitudinal and transverse magnetic fields we use the scaling property. The longitudinal and transverse magnetic fields in 2D approximations are connected to one another through 1D HT.

Equation (3) and equation (4) shows the longitudinal magnetic fields and transverse magnetic fields respectively in terms of magnetic charge representation of the magnetic media.

$$Hx(x) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{Hy(x')}{x+x\prime} dx' \quad (3)$$

$$Hy(x) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{Hx(x')}{x-x\prime} dx' \quad (4)$$

By using the scaling property of the Fourier transform, we can work to deduce it in terms of Hilbert Transform. By the help of magnetic recording, it is useful to find the Fourier representation of the 2D Hilbert transforms. HT can be formally extended to n dimensions, in the n dimensional extension the HT will be understood as an integral transform that relates the normal derivative of a harmonic function to the same harmonic function on the flat boundary.

B. HT in designing of Filters.

- By considering the HT equation in the 1 D and 2 D domains we design a filter such as a Digital all pass filter, Low pass filter and notch filter. For this application we use the discrete Hilbert transform that can be a relation between the log magnitude and phase response of minimum phase transfer function.
- In this we try to introduce the relation of the 1 D and 2 D discrete Hilbert transform for complex signals as an approximated form that also uses the discrete Fourier Transform.
- For an all pass filter design we consider its unique property of varying only phase with constant magnitude or it can be used as a phase compensator for distorted signals. We also consider the design methods that use the HT.
- For a low pass filter design we consider the use of HT in digital filtering to that of a matrix multiplication. The various matrix formulations for low pass filter (LPF) which requires less storage and reduces the number of multiplication by a factor of 2. In the formulations we exhibit two types of error i.e. quantization and round off error. The result shows that the new formulations have computational advantages and better noise characteristics.

C. HT in Image Processing.

The various television images that are of continuous value are transmitted using modulation techniques for video and audio signals. HT acts as a promising algorithm also for the earth images.

HT is considered to be useful in manipulating images since the transmission bandwidth is efficiently reduced. And the HT is also proposed as one of the many coding techniques that can be used in practical fields for the imaging materials.

For the image processing we consider the analytical function as the sum of the real and imaginary parts in terms of x and y nominations of the images as they are considered for the 2D domain. For example we take the Fourier transform algorithm which is often applied to the same number sequences in the time and frequency domains.

Standstill image signals are non fluctuations materials and information. So the digitized and stored continuous images for evaluations and verifications can be derived from continuous images at one instant of time. HT is used for the continuous signals for the images that adopt such algorithm in modulation systems particularly.

D. HT in Edge Detection.

- Edges represent the discontinuities in the intensity in an image. Edges created by occlusions, shadows, roofs, textures, etc. may have the coherent local intensity. Edge detection is a process that measures, detects, and localizes the changes in intensity. Edge detection is an important step in the process of segmentation also.
- In this a new method for edge detection using one dimensional processing is used which is the Gaussian function. The image is smoothed using 1 D Gaussian along the horizontal (or vertical) scan lines to reduce noise. Detection is then used in the orthogonal direction i.e., along vertical (or horizontal) scan lines to detect the edges.
- This method is based on the 2 D operators in the sense that smoothing is done along one direction in the sense that smoothing is done along one direction and the detection is applied along the orthogonal direction. But it also results in some loss of edge information.

CONCLUSION

In this survey paper we studied in detail about the Hilbert Transform by making use of its various mathematical formulations. It is a very important analytical tool that is used for the various applications. The application of the HT is studied and discussions. Study of all applications and there different ways by which HT is used for signal processing. This survey provides help to researchers for getting the detailed knowledge about the HT and its wide range of scope.

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